Algorithm for Calculating the Sensory Effect of a Three-Layer Optical Scheme by the Matrix Transfer Method

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Summary

A three-layer optical system "prism - gold nanolayer – dielectric" was investigated for the effect of the refractive index of the prism (first layer) and the permittivity of the dielectric (third layer) with the constant parameters of the gold nanolayer (second layer) on the reflection coefficient of the system under consideration. The studies were carried out accurately in view of the modeling of a real experiment by the transfer matrix method. The dependences of the reflection coefficient of the three-layer system on the wavelength λ at different values of the varied parameters were obtained.

1. Introduction

The transfer matrix method is used to calculateepy complex optical sensor systems. This method, on the one hand, allows calculate rather complex (multilayer) systems, although it is an approximate method. And, on the other hand, it is quite accurate and allows to actually simulate a real experiment in sens of the resonant wavelength and resonant incidence angle values finding.

2. Formulation of the problem

The matrix transfer method is based on the use of two types of 2×2 matrices [1]. These are the dynamic matrix $D_{\alpha,\beta}$, which describes the processes of light reflection and transmission at the interface between layer with number α and layer with number β, and the propagation matrix P_α, which determines the propagation of direct and reflected waves in layer with number α:

$$D_{\alpha,\beta} = \frac{1}{t_{\alpha,\beta}}\begin{pmatrix} 1 & r_{\alpha,\beta} \\ r_{\alpha,\beta} & 1 \end{pmatrix}; \qquad P_\alpha = \begin{pmatrix} \exp(-i\,k_\alpha d_\alpha) & 0 \\ 0 & \exp(i\,k_\alpha d_\alpha) \end{pmatrix} \tag{1}$$

In these definitions, the coefficients of reflection $r_{\alpha,\beta}$, transmission $t_{\alpha,\beta}$ and propagation k_α are determined by the Fresnel and Airy formulas [2] and have the form:

$$r_{\alpha,\beta} = \frac{\varepsilon_\beta\, k_\alpha - \varepsilon_\alpha\, k_\beta}{\varepsilon_\beta\, k_\alpha + \varepsilon_\alpha\, k_\beta}; \quad t_{\alpha,\beta} = \frac{2\varepsilon_\alpha\, k_\beta}{\varepsilon_\beta\, k_\alpha + \varepsilon_\alpha\, k_\beta}; \quad k_\alpha = \frac{2\pi}{\lambda}\sqrt{\varepsilon_\alpha - n_1^2 \sin\left(\frac{\pi\alpha}{180}\right)} \tag{2}$$

Here n_1 corresponds to the refractive index of the prism n_p, and angle α is the incidence angle of light (in degrees).

A typical three-layer optical scheme "prism - gold nanolayer – dielectric" was considered and the effect of two factors (the refractive index of the prism (first layer) and the permittivity of the dielectric (third layer)) on the resonant wavelength and resonant angle of incidence (the main sensory characteristics) was investigated. The parameters of the gold nano-layer (the second layer) were assumed to be constant.

The resonant mode at the calculations was selected from the condition of the maximum value of the ratio of the depth of the reflection coefficient minimum to its half-width and from the condition of the maximum sensitivity of the refractive index value to the change in the permittivity of the dielectric (third layer).

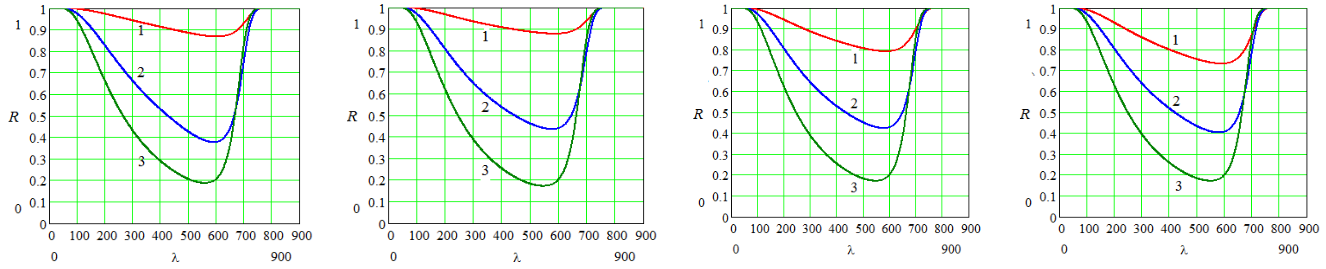
3. Conducted Research

The calculations were performed in the widest possible range of prism refractive indices n_p ={1.44; 1.55; 1.66; 1.77; 1.88; 1.99; 2.10; 2.21; 2.32; 2,42}. These values cover almost the entire range of refractive indices for optical glasses used in prisms (the first layer). And even includes some exotic values [3], such as 2.42 for diamond.

For permittivity values of the dielectric ε (третій шар) three values typical in the studies were chosen: ε={1.742; 1.784; 2.341}. The first two of them correspond to water in the temperature range from 0°C to 100°C [4], and the third value corresponds to a layer of DNA molecules used in biosensors [3].

4. Results

Figure 1 shows some (12 out of 30) of the results of the reflection coefficient calculations for the research system.



a *b* *c* *d*

Figure 1. Dependence of the reflection coefficient *R* of the three-layer system "prism - gold nano-layer - sensor dielectric" on the wavelength λ (nm). The numbering of the curves in each of the four graphs corresponds to the following values of the permittivity ε of the sensor dielectric: curves 1 - correspond to $\varepsilon = 1.742$; curves 2 - correspond to $\varepsilon = 1.784$; curves 3 - correspond to $\varepsilon = 2.341$. Graphs *a* - correspond to the values $n_p = 1.44$, $\alpha_{res} = 66.4°$; graphs *b* - correspond to the values $n_p = 1.77$, $\alpha_{res} = 48.2°$; graphs *c* - correspond to the values $n_p = 2.10$, $\alpha_{res} = 38.9°$; graphs *d* - correspond to the values $n_p = 2.42$, $\alpha_{res} = 33.0°$.

References

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